# Influence of non-uniform $J_{\rm c}$ distributions on flux jumps in high-temperature superconductors

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**Abstract.** The influence of non-uniform  $J_c$  distributions on flux jumps in high-temperature superconductors is investigated with the simplified models in the form of a composite superconducting slab consisting of different pinning regions. The magnetization loops and flux jumps for the simplified models are calculated with the modified recursion formulas which could predict the flux-jump fields. The valid ranges of the temperature and sweep rate for  $B_{fj1}$  are specified and the fields of the subsequent jumps after the first flux jump of the inhomogeneous models are obviously lower than that of the homogeneous models. The results indicate that flux jump instability could be stimulated by non-uniform  $J_c$  distributions and with the increase of the discrepancy of  $J_c$  between different pinning regions the instability also increase.

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#### **1** Introduction

Magnetic flux jumps are one of the peculiar phenomena of interest in both hard type- II superconductors and in hightemperature superconductors (HTS's). The investigation of flux jumps in HTS's is relevant to understanding the complexity of the vortex matter in the mixed phase of these materials. It is known that under appropriate conditions the critical state of a superconductor may become unstable, leading to an avalanchelike process, initiated by a small fluctuation of either the external magnetic field or the temperature. The process is associated with the sudden puncture of magnetic flux into the volume of the superconductor with a corresponding increase in the material's temperature. During this process, the screen current is appreciably reduced, perhaps even to zero. Thus, flux jumps are problematic as they may drive the superconductor into a normal or resistive state. Since flux jumps are undesirable in practical applications of superconductors, this phenomenon has been widely studied [1,2].

The basic theory appropriate to magnetic flux jumping was developed in the late 1960s by Swartz et al. [3]. Magnetic stability predictions for HTS have been made ever since the necessary parameters became available, which are the critical temperature, the sweep rate and the specific heat [4]. After a criterion of disturbance of the magnetic field was employed based on the Kim-Anderson model, Müller and Andrikidis [5] deduced an analytical formula which could predict the first flux-jump field, but more importantly, they obtained the recursion formulas which could predict the fields of subsequent jumps after the first flux jump. To reflect the effect of sweep rate on the flux-jump field in the theoretical analysis, on the other hand, Mints [6] proposed a theoretical model to study flux jumping based on critical state models in the fluxcreep regime of type-II superconductors. Then, Nabialek et al. [7] investigated magnetic flux jumps in textured BiSrCaCuO by means of magnetization measurements in the temperature range between 1.95 K and  $T_c$ . In addition, Zhou et al. [8] studied the influence of some parameters on the flux-jump field by numerical simulations. Recently the influence of non-uniform pinning potential on the local flux creep in type-II superconductors was studied by numerical simulations and the conclusion that flux profiles inside the inhomogeneous superconductors are obviously different from the homogeneous ones was obtained [9]. Moreover, the influence of locally varying  $J_c$  value on AC transport losses in self-fields is investigated on superconductor wires and tapes with various cross sectional geometry through numerical calculations, and it was found that the distribution of  $J_c$  could affect significantly the loss value [10]. Obviously the influence of non-uniform pinning potential on flux jumps is not considered in their work. Thus, the influence of non-uniform  $J_c$  distributions on flux jumps is of interest both from a basic point of view and also in light of their potential applications.

In this paper, we modified the recursion formulas which could predict the flux-jump fields and compare the behavior of the homogeneous and inhomogeneous models with respect to the change of the temperature and sweep rate, then studied the influence of non-uniform  $J_c$  distributions on magnetization and flux jump instability in the case of no flux jumps and flux jumps respectively.

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## 2 Calculation method

On the basis of the approach to the flux jumping problems suggested by Müller and Andrikidis [5], the simulations of the magnetization loop and flux jumps in this paper were carried out by considering the change of temperature in the superconductor, especially by modifying the recursion formulas of predicting the fields of the following jumps after the first flux jump. Under the assumption of local adiabatic condition, the heat exchange between different pinning regions is also neglected. The approach of Müller and Andrikidis combined the predictions of the flux-jump fields and the calculations of magnetization on the basis of the critical state models, and the prerequisite of the approach is accord with the assumption of the simplified models in the paper.

In the approach suggested by Müller and Andrikidis, after employing the criterion of magnetic field disturbance  $\Delta H_1 = \Delta H_2$ , an analytical formula of predicting the first flux-jump field was deduced based on the Kim-Anderson model

$$B_{fj1} = \sqrt{2\mu_0 c J_c / (-dJ_c/dT)},$$
 (1)

where c is specific heat,  $J_c$  is critical current density, and  $\mu_0$  and T are the magnetic permeability of vacuum and temperature, respectively. The recursion formulas are given as

$$B_{j0}^{2} = 2((|B_{fj}| - |B_{h}|)(|B_{fj}| + B_{0}) + (|B_{fj}| + B_{0})(|B_{h}| + B_{0})\ln((B_{0} + |B_{h}|)/(B_{0} + |B_{fj}|))),$$
(2a)

$$B_{j0}^{2} = 2(|B_{fj}| + B_{0})((|B_{fj}| - B_{0})\ln((B_{0} + |B_{fj}|)/B_{0}) + (|B_{h}| + B_{0})\ln((B_{0} + |B_{h}|)/B_{0}) + |B_{fj}| - |B_{h}|)$$
(2b)

where  $B_0$  is a phenomenological parameter, the value of the field  $B_h$  depends on the magnetic history of the sample, for the first virgin jump the field  $B_h$  is equal to zero and for further jumps  $B_h$  is greater than zero. The absolute values of  $B_{fj}$  and  $B_h$  are introduced in equation (2a) to include the case where  $B_{fj} < 0$  and  $B_h < 0$ , and they are introduced in equation (2b) to include the case where  $B_{fj} > 0$  and  $B_h < 0$ . Replacing  $B_{j0}$  in equation (2a) by  $B_{fj1}$  obtained by equation (1) and substituting  $B_h$ into equation (2a) or (2b), we can obtain the fields of flux jumps. Then, from the analytical expressions of the magnetic-field profiles B(x), the magnetization M can be calculated as a function of the applied field  $B_a$  when  $B_a$ was swept between the maximum field  $B_m$  and the largest negative field  $-B_m$ . For a slab of thickness d

$$\mu_0 M = \frac{2}{d} \int_0^{d/2} B(x) dx - B_a, \qquad (3)$$

however, during the derivation process suggested by Müller and Andrikidis, the sweep rate of external magnetic field was not considered, and it is assumed that each flux jump is incomplete and that the maximum temperature reached during a jump is the same for all jumps. Actually flux jumps depends significantly on the sweep rate, and the assumption that each jumps is incomplete and the maximum temperature reached during a jump is the same for all jumps could be neglected. Thus, some amendments were carried out. First, the influence of the sweep rate on flux jumps was reflected in the recursion formulas by introducing the relation of  $B_{fj1} \sim \dot{B}_e^{-1/3}$  deduced by Mints [6] in terms of the Kim-Anderson model, equation (1) was then modified as

$$B_{fj1} = \left(2\mu_0 c J_c / (-dJ_c/dT)\right)^{1/2} * \dot{B}_e^{-1/3}, \qquad (4)$$

second, because a flux jump occurs at each temperature jump, the change of temperature can be obtained by estimating the magnetization hysteresis loss during each jump [11]

$$W = \int dW = \int M dH, \tag{5}$$

then, by integrating the heat capacity between the temperature before the jump (i.e., the temperature  $T_0$ ) and its maximum temperature during the jump  $T_f$ , the heat absorbed by the sample during the flux jump is obtained as

$$Q = \int c(T)dT.$$
 (6)

For the heat capacity c(T) the experimental measurement shows that it is formulated by  $c(T) = \gamma T^3$ . For the Bi samples it is found that  $\gamma \approx 14.8 \text{ J/K}^4 \text{ m}^3$ , and then we get the following equation

$$Q = \gamma \left( T_f^4 - T_0^4 \right) / 4,\tag{7}$$

by equating the dissipative work with the absorbed heat, the value of  $T_f$  which represents the maximum temperature during a flux jump could be obtained. During a jump, corresponding to the maximum temperature  $T_f$  the critical current density  $J_{0j}$  is given by fitting the experimental data with relation  $J_c(T,0) =$  $J_{c0} \exp(-T/[T_e(1-(T/T_c)^2)])$ , in which  $T_e = 8.4$  K and  $T_c = 92$  K.

#### 3 Results and discussion

We consider a semi-infinite composite superconducting slab of thickness d consisting of two pinning regions with different values of  $J_c$ . The configuration of the slab and field direction is shown in Figure 1. In this configuration, parameters representing properties of the weak or strong pinning region are denoted by subscripts w and s, respectively. The strong region is characterized by a relatively high critical current density  $(J_{cs})$  and the weak one is a relatively low critical current density  $(J_{cw})$  in the simplified models. If the weak region is in front of the strong one we call it the WS model, on the contrary, the SW model is called. In particular, if  $J_{cs} = J_{cw}$ , i.e.,  $\beta = 1$ , we call it the WU or SU model corresponding to the magnitude of  $J_c$ . For further analysis, the ratio  $\beta = J_{cs}/J_{cw}$  is introduced to characterize the discrepancy of  $J_c$  between



Fig. 1. Schematic sketch of the simplified superconducting models consisting of strong and weak pinning regions denoted by subscripts s and w, respectively.



Fig. 2. (a) The prediction curve of the first flux-jump field versus the temperature for the present paper compared with the experimental and numerical curves, respectively. (b) The prediction curve of the first jump field versus the sweep rate of external magnetic field for the present paper compared with the experimental and numerical curves.

different pinning regions since the diffusion and distribution of magnetic field and heat in superconductors may be influenced by  $\beta$ , and the same dependence of  $J_{cw}$  and  $J_{cs}$ on external magnetic field is assumed.

Figure 2 shows the dependence of the temperature and sweep rate on  $B_{fj1}$ . A comparison of the predictions with the experimental data given in reference [7] and the numerical simulations given in reference [8] and the modified



Fig. 3. The magnetization loop with flux jumps simulated for the homogeneous model when the temperature is 4.2 K and the sweep rate is 50 G/s.

formula of equation (4) are displayed. From Figure 2a, one finds that the results of the numerical simulations and equation (4) are higher than the experimental data only between about 3.8 and 5.5 K, and the predictions of equation (4) are accord with the numerical simulations and experimental data when the temperature is relatively low. With the increase of temperature, the first flux-jump field raised rapidly at temperature above 5.5 K, both the numerical simulations and equation (4) is not suitable to predict the fields of flux jumps. Thus, for temperature between 3 and 5.5 K, the predictions of equation (4) are valid. In fact, the predictions of equation (4) for the dependence of the sweep rate on  $B_{fj1}$  are related to the rationality of the amendments in this paper. Figure 2b shows that the predictions of the numerical simulations and equation (4) are extremely similar within the entire interval of the sweep rate, and they are lower than experimental data when the sweep rate is below 50 G/s and higher than experimental data when the sweep rate is above 50 G/s. As the sweep rate increases from 20 G/s to 50 G/s, the experimental data steeply decreases. With the increase of sweep rate from 50 G/s, the results of equation (4) are closer to the experimental data compared with the numerical simulations and  $B_{fj1}$  of equation (4) tend to a saturation value of about 1 T. Therefore, when the sweep rate exceeds 20 G/s it is thought that the predictions of equation (4) are valid.

Figure 3 illustrates the simulations of the magnetization loop with flux jumps in terms of the modified approach. In the simulations, the external magnetic field is swept from 0 to 9 T, back to -9 T and again back to zero with a specified sweep rate of  $\dot{B}_e = v_{ex} = 50$  G/s and a temperature of 4.2 K. Some parameters are taken as  $J_{c0} = 3.0 \times 10^{10}$  A/m<sup>2</sup>, and we use the approximate relations of  $c = 14.8T_0^3$ ,  $\alpha(T_0) \approx J_{c0}B_0e^{-T_0/T_e}$  and  $B_0 = 0.3$  T. According to the experimental data, at temperature above 3 K all of the observed jumps are complete and the third quadrant of the magnetization loops is the most unstable in all quadrants. From this figure, one sees that in the third quadrant the number of flux jumps is



Fig. 4. When the temperature and sweep rate is in turn specified, the influence of the temperature and sweep rate on the inhomogeneous models with flux jumps is shown at the first quadrant of the magnetization loops.

the most and almost all flux jumps are complete. In other word, the results observed in the figure are basically accord with the experimental and numerical data.

It has been known that for the homogeneous superconductors flux jumps depends strongly on the changes of both the temperature and sweep rate [7,8]. When the sweep rate is specified, with the increase of temperature the jump fields and the field spacing between subsequent jumps increase, but the number of jumps decreases. Similarly, when the temperature is specified, with the increase of sweep rate the jump fields and the field spacing decrease, but the number of jumps increases. In order to reveal the influence of the temperature and sweep rate on flux jumps in the inhomogeneous models, by in turn specifying the temperature of 4.2 K and sweep rate of 50 G/s, the virgin magnetization loops with flux jumps for the inhomogeneous models in which  $J_{cw}$  is  $1.0 \times 10^{10} \text{A/m}^2$  and  $\beta = 3$  are shown in Figure 4. Of course,  $\beta$  could be arbitrarily specified. The results show that non-uniform  $J_c$ distributions did not change the above mentioned features and for the SW model the influence of the temperature and sweep rate on flux jumps is more obvious compared with the WS model (see Figs. 4c and 4d).

Figure 5 shows the influence of  $\beta$  on the magnetization loop without flux jumps calculated in terms of the Bean and Kim-Anderson models. The non-uniformity of the simplified models is reflected by the change of  $\beta$ . In particular,  $\beta = 1$  means that the distribution of  $J_c$  in the composite slab is uniform. We emphasize that the nonuniformity of models is also enhanced with the increase of  $\beta$  and when flux jumps do not occur  $\beta$  is only important to the distribution and diffusion of magnetic field. According to Figure 5, with the raise of  $\beta$  the ingress and out of flux lines for the strong region is more and more difficult and the fields at which the magnetization approaches the saturation value become more and more high. We also note that the shape of magnetization loops become more and more strange, especially in the third quadrant. Moreover, the influence of  $\beta$  on the SW model is also more obvious than on the WS model.

Figure 6 shows the influence of  $\beta$  on the magnetization loops with flux jumps simulated on the process of thermomagnetic interaction. As the temperature of 4.2 K and sweep rate of 50 G/s are specified,  $B_{fj1}$  and the number of flux jumps is also determined. From Figures 6a and 6b, when  $\beta = 3$  and  $\beta = 7$ , except  $B_{fj1}$ , the other jump fields of the inhomogeneous models are obviously lower than that of the homogeneous ones and the phenomenon for the WS model is obvious compared with the SW model. Furthermore, we find that this trend becomes more and more evident with the continued flux jumps. In order to explain the reason of the phenomenon, the calculation method presented by the paper is examined in detail. It is found that during each jump for the WS



Fig. 5. The magnetization loops without flux jumps calculated at T = 4.2 K with critical state models for the inhomogeneous models when the ratio of  $\beta = J_{cs}/J_{cw}$  is changed from 1 to 7. Figures 5a and 5b are calculated with the Bean model, the other are calculated with the Kim-Anderson model.

model the sum of the energies released in both the weak and strong region is always smaller than the SU model and sometimes even smaller than the WU model. This may be the real reason that the flux jump instability of the inhomogeneous models was increased. From the other figures, one sees that with the increase of  $\beta$  from 1 to 7 the jump fields gradually decrease and this case for the WS model is relatively obvious compared with the SW model, meaning that the change of  $\beta$  clearly affect flux jump stability of the inhomogeneous models.

## 4 Conclusion

We investigate the influence of non-uniform  $J_c$  distributions on the magnetization loops and flux jumps instability in the composite superconducting slab consisting of different pinning regions on the basis of the simplified models proposed by this paper. The recursion formulas for predicting the fields of flux jumps are modified by considering the effect of the sweep rate and the ratio of  $\beta$  is introduced to characterize the discrepancy of  $J_c$  between different pinning regions. The magnetization loops and flux jumps for the homogeneous and inhomogeneous simplified models are calculated with the change of  $\beta$ . The valid ranges of the temperature and sweep rate are specified for the predictions of  $B_{fj1}$  by the modified formulas. The influence of the temperature and sweep rate on flux jumps exhibits the same behavior for the homogeneous and inhomogeneous models. The jump fields except  $B_{fj1}$  of the inhomogeneous models are obviously lower than that of the homogeneous ones and this phenomenon for the WS model is more obvious compared with the SW mode and the trend become more and more evident with the continued flux jumps. The results indicate that flux jump instability could be stimulated by non-uniform  $J_c$ distributions and with the increase of  $\beta$  the instability are increased. The influence of non-uniform  $J_c$  distributions not only on the magnetic relaxation and AC transport losses but also on flux jumps is suggested.

However, it should be noted that the inhomogeneous models in this paper are simple and under the assumptions of local adiabatic condition the heat exchange between different pinning regions is neglected. By developing the inhomogeneous models whose  $J_c$  linearly increases from the edge to the center of the models, and by considering the heat exchange including between different pinning regions and between the model and coolant, we believe that the comprehensive understanding of this phenomenon will be accomplished.



Fig. 6. The virgin magnetization loops with flux jumps for the homogeneous and inhomogeneous models with different  $\beta$  when the temperature is 4.2 K and the sweep rate is 50 G/s. Figure 6a and 6b show a comparison of the homogeneous and inhomogeneous model when  $\beta$  is 3 and 7, respectively. With the increase of  $\beta$  from 1 to 7, Figures 6a and 6b show the comparison of magnetization loops of the inhomogeneous model including the WS and SW model, respectively.

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